UNIVERSITY OF OTTAWA
Principles of Physics
PHY1321/1331 FALL 2011
Dr. A. Czajkowski

CLASS: PHY_____

STUDENT #: ________________

NAME: ______________________

Assignment 2: KINEMATICS 2-D Motion

Assigned: Sept 16 14:30 Due: September 23 19:00

1 By algebraic manipulation of the first two kinematic equations for one-dimensional motion:

$$1)v_f = v_i + at \qquad 2)x_f = x_i + v_i t + \tfrac{1}{2}at^2$$

Obtain the other two kinematic equations: $3)v_f^{\,2} - v_i^{\,2} = 2a\Delta x \qquad 4)x_f = x_i + \tfrac{1}{2}(v_i + v_f)t$

SOLUTION:

$$1)v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)x_f = x_i + v_i\frac{(v_f - v_i)}{a} + \tfrac{1}{2}a\frac{(v_f - v_i)^2}{a^2} \Rightarrow x_f - x_i = v_i\frac{(v_f - v_i)}{a} + \tfrac{1}{2a}\left(v_f^{\,2} - 2v_iv_f + v_i^{\,2}\right) \Rightarrow$$

$$\Rightarrow 2a(x_f - x_i) = 2v_i(v_f - v_i) + \left(v_f^{\,2} - 2v_iv_f + v_i^{\,2}\right) \Rightarrow 2a(x_f - x_i) = 2v_iv_f - 2v_iv_i + v_f^{\,2} - 2v_iv_f + v_i^{\,2} \Rightarrow 2a(x_f - x_i) = v_f^{\,2} - v_i^{\,2}$$

$$1)v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)x_f = x_i + v_i t + \tfrac{1}{2}a\frac{(v_f - v_i)}{a}t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2}(v_f - v_i)t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2}v_f t - \tfrac{1}{2}v_i t \Rightarrow x_f = x_i + \tfrac{1}{2}(v_f + v_i)t$$

2 A test rocket is fired vertically upward from a well. A catapult gives it initial velocity 60.0 m/s at ground level. Its engines then fire and it accelerates upward at 5.00 m/s^2 until it reaches an altitude of 1 000 m. At that point its engines fail and the rocket goes into free fall, with an acceleration of –9.80 m/s^2. (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion)

$$1)v_f^{\,2} = v_i^{\,2} + 2a\Delta h \Rightarrow v_f^{\,2} = 60^2 + 2(5)(1000) = 13600 \Rightarrow v_f = 116.6(\tfrac{m}{s})$$

$2) \Rightarrow x_f = x_i + \tfrac{1}{2}(v_f + v_i)t = 0 + \tfrac{1}{2}(60 + 116.6)t = 88.3t \Rightarrow 1000 = 88.3t = 11.32s$ rocket acceleration time

3) $0 = 1000 + v_i t + \tfrac{1}{2}at^2 \Rightarrow 0 = 1000 + 116.6t - \tfrac{1}{2}9.8t^2 \Rightarrow t = 30.39s$ rocket free fall time

a) Total time in the air 41.7 s

4) $v_f^{\,2} = v_i^{\,2} + 2a\Delta h \Rightarrow 0 = 116.6^2 - 2(9.8)(\Delta h) = 13 \Rightarrow \Delta h = 693.6m$ this is the max elevation of the rocket with respect to the beginning of the free fall motion (when h=1000)

b) Maximum elevation is 1694m

5) $2) \Rightarrow 0 = 1000 + \tfrac{1}{2}(116.6 + v_f)41.7 \Rightarrow v_f = -164.6(\tfrac{m}{s})$

c) The final speed is 165m/s

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Assignment 2: KINEMATICS 3-D Motion CONT

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3 An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. If *x* is horizontal and *y* vertical, find the (*x*, *y*) coordinates where the shell explodes.

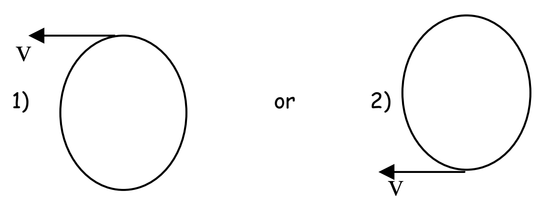
$$x_f = x_i + v_x t = x_i + v_0(\cos\theta)t = 0 + 300(\cos 55)42 = 7227m$$

$$y_f = y_i + v_y t + \tfrac{1}{2}a_y t^2 = 0 + v_0(\sin\theta)t - \tfrac{1}{2}gt^2 = 0 + 300(\sin 55)42 - 4.9(42)^2 = 1677m$$

ANS: 7.23 km, 1.68 km

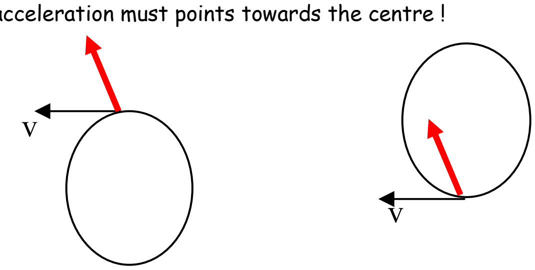
4. A car travels in a flat circle of radius R. At a certain instant the velocity of the car is 24 m/s west, and the total acceleration of the car is 2.5 m/s^2 53° north of west. Find the radial and tangential components of the acceleration of the car. How long will it take for the car to make a one full circle from the point at which its velocity is 24m/s west?

It seems that the information about the position is missing. The only possible realizations of the described situation from the point of view of the velocity direction are these two:



since the velocity is always tangent to the trajectory (circle)!!!

If the direction of the acceleration is considered it becomes clear that only ythe second position is physically possible. (radial component of the acceleration must points towards the centre !



Impossible due to the direction of acceleration vector!

this is our case!

The rest of the solution follows the same lines as the problem solved in class.

$$a_r = 2.5\frac{m}{s^2}\sin 53 = 2.00 \qquad a_t = 2.5\frac{m}{s^2}\cos 53 = 1.50$$

There is also no information about radius given directly, but we can obtain it from v and radial component of acceleration:

$$a_r = \frac{v^2}{r} \Rightarrow \quad r = \frac{v^2}{a_r} = 288(m)$$

Using a_t, v and r in kinematic equation

$$2\pi r = vt + \tfrac{1}{2}a_t t^2 \Rightarrow 1810 = 24t + 0.75t^2$$

$$\boxed{t = 35.7s}$$

5 The projectile motion is fired with velocity of magnitude v_0 at the angle Θ. Find Θ for which the maximum elevation of the projectile is twice its range.

We will use the equation for projectile motion trajectory y(x) for x=R/2 / 2R=y(R/2)

$$y = (\tan\theta)x - \frac{g}{2(v_0\cos\theta)^2}x^2 \Rightarrow 2R = (\tan\theta)(\frac{R}{2}) - \frac{g}{2(v_0\cos\theta)^2}(\frac{R}{2})^2 \Rightarrow 2 = (\tan\theta)(\frac{1}{2}) - \frac{g}{2(v_0\cos\theta)^2}\frac{R}{4}$$

$$2 = (\tan\theta)(\frac{1}{2}) - \frac{g}{2(v_0\cos\theta)^2}\frac{v_0^{\,2}\sin 2\theta}{4g} \Rightarrow 2 = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{8}\frac{\sin 2\theta}{\cos^2\theta} \Rightarrow 2 = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{8}\frac{2\sin\theta\cos\theta}{\cos^2\theta}$$

$$2 = \frac{1}{2}\frac{\sin\theta}{\cos\theta} - \frac{1}{4}\frac{\sin\theta}{\cos\theta} \Rightarrow 8 = \tan\theta \Rightarrow \theta = 82.9^o$$

$$\boxed{\theta = 82.9^o}$$